Say whether the following ode's are linear or nonlinear (L or NL),
exact or not (E or NE), separable or not (S or NS).

<table>
<thead>
<tr>
<th></th>
<th>Equation</th>
<th>L or NL</th>
<th>E or NE</th>
<th>S or NS</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>$t^2 \frac{dy}{dt} + ty = t$</td>
<td></td>
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<tr>
<td>2</td>
<td>$3t = e^t \frac{dy}{dt} + y \ln t$</td>
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<td>3</td>
<td>$y^2 , dx = -(2xy + \cos y) , dy$</td>
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<tr>
<td>4</td>
<td>$y , dx = x , dy$</td>
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<td>5</td>
<td>$3r = \frac{dx}{dt} - \theta^3$</td>
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<tr>
<td>6</td>
<td>$\frac{dy}{dx} = \frac{3x^2 + 4x + 2}{2y + 1}$</td>
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<tr>
<td>7</td>
<td>$\frac{d^2 y}{dx^2} + ty \frac{dy}{dt} = 0$</td>
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<tr>
<td>8</td>
<td>$\frac{dy}{dx} = 5$</td>
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Solve the initial value problem

$$\frac{dy}{dx} - 2xy = 0 \quad y(0) = 1$$

$$y(x) =$$

*Hint. You will find $y(\sqrt{\ln 2}) = 2$*
Solve the following differential equations

\[(x + 1) \, dy = (\cos x - y) \, dx\]

\[\frac{dv}{dx} = \frac{1 - 4v^2}{3v}\]

\[y' = x^3(1 - y), \quad y(0) = 3\]
\[ \frac{dy}{dx} = x - 2xy \]

\[ \cos x \frac{dy}{dx} = -y \sin x + 2x \cos^2 x \]

\[ (1/y) \, dx + (3y - x/y^2) \, dy = 0 \]
The population $P(t)$, measured in units of 1000, of a species obeys the ode

$$\frac{dP}{dt} = 2P(1 - P)$$

where time $t$ is measured in years.

If the starting population was 2000, i.e. $P(0) = 2$,

To which value does $P(t)$ tend as $t \to \infty$?

$$\lim_{t \to \infty} P(t) =$$
The amount of radioactive isotope \( Q(t) \) (measured in kgs. or as a percentage of its original value) obeys the ode

\[
\frac{dQ(t)}{dt} = -kQ(t)
\]

where \( t \), time is measured in years. Given \( Q(0) = Q_0 \), find \( Q(t) \) in terms of \( Q_0, k \) and \( t \).

\[
Q(t) = \ldots
\]

The half-life \( \tau \) is the time it takes the amount \( Q(t) \) to decay to \( \frac{1}{2}Q_0 \); i.e. \( Q(\tau) = \frac{1}{2}Q_0 \). Find an expression for \( \tau \) in terms of \( k \), the decay rate

\[
\tau = \ldots
\]

A bone is found on an archeological dig. When found, the amount of radioactive isotope \( ^{14}\text{C} \) (carbon 14) was 25% of what it was when the bone was part of a living organism. Given the half life of \( ^{14}\text{C} \) is 5600 years, how long has the bone been dead (age)?

\[
\text{Age} = \ldots
\]
Consider the ode

\[ \frac{dy}{dt} = y(1 - y)(2 - y) \]

Given (a) \( 0 < y(0) < 1 \), \( y(t) \to \) \[\square\] as \( t \to \infty \).

Given (b) \( 1 < y(0) < 2 \), \( y(t) \to \) \[\square\] as \( t \to \infty \).

Given (c) \( y(0) > 2 \), \( y(t) \to \) \[\square\] as \( t \to \infty \).

Which of the equilibrium points \( y = 0 \), \( y = 1 \), and \( y = 2 \) is stable?
Model for an object falling toward earth. Assuming that only air and gravity are acting on the object, the velocity $v$ is given by

$$m \frac{dv}{dt} = mg - bv$$

where $m$ is the mass of the object, $g$ is the acceleration due to gravity and $b > 0$ is constant. If $m = 100\, kg$, $g = -9.8\, m/sec^2$, $b = 5\, kg/sec$, $v(0) = 10\, m/sec$, solve for $v(t)$. What is the terminal velocity?
Newton’s law of cooling:

\[
\frac{dH}{dt} = k(T - H)
\]

where \( H \) is the temperature of the object, \( k > 0 \) is constant, and \( T \) is the temperature of the surrounding.

Solve for \( H \).

A thermometer reading 100°F is placed in a medium having a constant temperature of 70°F. After 6 minutes, the thermometer reads 80°F. What is the reading after 20 minutes?
Consider the following non-homogeneous equations. What is the form of a particular solutions? (you do not need to find the value of the coefficients). The particular solution found using the undetermined coefficient method.

a) $y'' + 25 y = \cos (3 t)$

b) $2 y'' + 128 y = \sin (8 t)$

c) $y'' + 2 y' - 3 y = 2 t e^t \sin t$

d) $y'' + 2 y' - 3 y = 5 t^2 e^{-3 t}$

e) $y'' - 4 y' + 13 y = \cos (3 t) + e^{2 t}$

f) $y'' - 4 y' + 13 y = t^2 e^{2 t} \cos (3 t) + \sin (3 t)$

g) $y'' - 6 y' + 9 y = t e^{3 t}$
A particular solution to
\[ y'' - 4y' + 13y = e^t \]
is given by \( y_p = \frac{1}{11} e^t \).

a) Find the general solution to the equation.

b) Find the solution that satisfies the conditions \( y(0) = 1 \) and \( y'(0) = 0 \).