Review for Exam #1 MATH 323

Say whether the following ode's are linear or nonlinear (L or NL), exact or not (E or NE), separable or not (S or NS).

	Equation	L or NL	E or NE	S or NS
1	$t^2 \frac{dy}{dt} + ty = t$			
2	$3t = e^t \frac{dy}{dt} + y \ln t$			
3	$y^2\, dx = -(2xy + \cos y)\, dy$			
4	$y\, dx = x\, dy$			
5	$3r = \frac{dr}{d\theta} - \theta^3$			
6	$\frac{dy}{dx} = \frac{3x^2+4x+2}{2y+1}$			
7	$\frac{d^2y}{dt^2} + ty\frac{dy}{dt} = 0$			
8	$\frac{dy}{dx} = 5$			

Solve the initial value problem

$$\frac{dy}{dx} - 2xy = 0 \qquad y(0) = 1$$

$y(x) =$

Hint. You will find $y(\sqrt{\ln 2}) = 2$

Solve the following differential equations

$$(x+1)\,dy = (\cos x - y)\,dx$$

$$x\frac{dv}{dx} = \frac{1-4v^2}{3v}$$

$$y' = x^3(1-y), \quad y(0) = 3$$

$$\frac{dy}{dx} = x - 2xy$$

$$\cos x \, \frac{dy}{dx} = -y \, \sin x + 2 \, x \, \cos^2 x$$

$$(1/y) \, dx + (3y - x/y^2) \, dy = 0$$

The population $P(t)$, measured in units of 1000, of a species obeys the ode

$$\frac{dP}{dt} = 2P(1-P)$$

where time t is measured in years.

If the starting population was 2000, i.e. $P(0) = 2$,

To which value does $P(t)$ tend as $t \to \infty$?

$\lim_{t\to\infty} P(t) =$

The amount of radioactive isotope $Q(t)$ (measured in kgs. or as a percentage of its original value) obeys the ode

$$\frac{dQ(t)}{dt} = -kQ(t)$$

where t, time is measured in years. Given $Q(0) = Q_0$, find $Q(t)$ in terms of Q_0, k and t.

$Q(t) =$

The half-life τ is the time it takes the amount $Q(t)$ to decay to $\frac{1}{2}Q_0$; i.e. $Q(\tau) = \frac{1}{2}Q_0$. Find an expression for τ in terms of k, the decay rate

$\tau =$

A bone is found on an archeological dig. When found, the amount of radioactive isotope C^{14} (carbon 14) was 25% of what it was when the bone was part of a living organism. Given the half life of C^{14} is 5600 years, how long has the bone been dead (age)?

$Age =$

Consider the ode

$$\frac{dy}{dt} = y(1-y)(2-y)$$

Given (a) $0 < y(0) < 1$, $y(t) \rightarrow$ [ ] as $t \rightarrow \infty$.

Given (b) $1 < y(0) < 2$, $y(t) \rightarrow$ [ ] as $t \rightarrow \infty$.

Given (c) $y(0) > 2$, $y(t) \rightarrow$ [ ] as $t \rightarrow \infty$.

Which of the equilibrium points $y = 0,\ y = 1$, and $y = 2$ is stable?

Model for an object falling toward earth. Assuming that only air and gravity are acting on the object, the velocity v is given by

$$m\frac{dv}{dt} = m\,g - bv$$

where m is the mass of the object, g is the acceleration due to gravity and $b > 0$ is constant. If $m = 100kg$, $g = -9.8m/sec^2$, $b = 5kg/sec$, $v(0) = 10m/sec$, solve for $v(t)$. What is the terminal velocity?

Newton's law of cooling:

$$\frac{dH}{dt} = k\left(T - H\right)$$

where H is the temperature of the object, $k > 0$ is constant, and T is the temperature of the surrounding.

Solve for H.

A thermometer reading 100°F is placed in a medium having a constant temperature of 70°F. After 6 minutes, the thermometer reads 80°F. What is the reading after 20 minutes?

Consider the following non-homogeneous equations. What is the form of a particular solutions? (**you do not need to find the value of the coefficients**). The particular solution found using the undetermined coefficient method.

a) $y'' + 25\,y = \cos(3\,t)$

b) $2\,y'' + 128\,y = \sin(8\,t)$

c) $y'' + 2\,y' - 3\,y = 2\,te^{t}\,\sin t$

d) $y'' + 2\,y' - 3\,y = 5\,t^2\,e^{-3\,t}$

e) $y'' - 4\,y' + 13\,y = \cos(3\,t) + e^{2t}$

f) $y'' - 4\,y' + 13\,y = t^2\,e^{2\,t}\cos(3\,t) + \sin(3\,t)$

g) $y'' - 6\,y' + 9\,y = t\,e^{3\,t}$

A particular solution to

$$y'' - 4\,y' + 13\,y = e^t$$

is given by $y_p = \frac{1}{10}\,e^t$.

a) Find the general solution to the equation.

b) Find the solution that satisfies the conditions $y(0) = 1$ and $y'(0) = 0$.