

Third review for the Final Exam

Mat 323, Fall 2005

1. Find the Laplace transforms of

(a) $f(t) = 2t^3 e^{2t}$

(b)

$$f(t) = \begin{cases} t^2 & \text{for } 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(c) $e^t u(t - 1)$

(d) $e^{-t} t \sin(2t)$

2. Find the inverse Laplace transforms of Find $f(t)$ for

(a) $F(s) = \frac{(3s + 5)e^{-s}}{s(s^2 + s - 6)}$

(b) $F(s) = \ln(s - 1)$ (hint: calculate the inverse Laplace transform of $F'(s)$ and use the last formula in the tables).

3. Solve the following differential equation

$$y'' + 4y' + 6y = g(t), \quad y(0) = 0, \quad y'(0) = 1$$

where

$$g(t) = \begin{cases} 1 & \text{for } 1 \leq t \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

4. Find the first four nonzero terms in a power series expansion about $x = 0$ for the solution to the initial value problem

$$(2x - 3)y'' - x y' + y = 0, \quad y(0) = 1, \quad y'(0) = 2$$

5. Find the recurrence relation for the power series solution (around $x = 0$) of

$$2y'' + xy' + 3y = 0.$$

Then use the recurrence to find the first 6 terms in the series for y .

System of equations: p. 541, problems 12,14,34; p. 549, problem 2

Tables

$f(t)$	$F(s)$
1	$\frac{1}{s}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s - a}$
$\cos(bt)$	$\frac{s}{s^2 + b^2}$
$\sin(bt)$	$\frac{b}{s^2 + b^2}$

Formulae
$\mathcal{L}(e^{at}f(t)) = F(s - a)$
$\mathcal{L}(f'(t)) = sF(s) - f(0)$
$\mathcal{L}(u(t - a)f(t - a)) = e^{-as}F(s)$
$\mathcal{L}(tf(t)) = -F'(s)$