Review for test 3

Mat 323, Spring 2005

1. Find the Laplace transforms of

(a) $f(t) = 2t^3 e^{2t}$

(b)

$$f(t) = \begin{cases} t^2 & \text{for } 0 \le t \le 1 \\ 0 & \text{otherwise} \end{cases}$$

(c) $e^t\, u(t-1)$

(d) $e^{-t}\, t\, \sin(2\,t)$

2. Find the inverse Laplace transforms of Find $f(t)$ for

(a) $F(s) = \dfrac{2\,s+3}{s^2+2s+2}$

(b) $F(s) = \dfrac{(3\,s+5)\,e^{-s}}{s\,(s^2+s-6)}$

(c) $F(s) = \dfrac{5s^2+34s+53}{(s+3)^2\,(s+1)}$

(d) $F(s) = \ln(s-1)$ (hint: calculate the inverse Laplace transform of $F'(s)$ and use the last formula in the tables).

3. Solve the following differential equation

$$y'' + 4y' + 6\,y = g(t), \quad y(0) = 0, \quad y'(0) = 1$$

where

$$g(t) = \begin{cases} 1 & \text{for } 1 \le t \le 2 \\ 0 & \text{otherwise} \end{cases}$$

4. If $\mathcal{L}(f(t)) = F(s)$, then find the Laplace transform $\mathcal{L}(tf'(t))$ in terms of s, $F(s)$ and $F'(s)$.

5. Find the first four nonzero terms in a power series expansion about $x = 0$ for the solution to the initial value problem

$$(2x - 3)y'' - x\,y' + y = 0, \quad y(0) = 1, \quad y'(0) = 2$$

Find a minimum value for the radius of convergence of the series.

6. Find the recurrence relation for the power series solution (around $x = 0$) of

$$2y'' + xy' + 3y = 0.$$

Then use the recurrence to find the first 6 terms in the series for y.

Tables

$f(t)$	$F(s)$
1	$\frac{1}{s}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\cos(b\,t)$	$\frac{s}{s^2+b^2}$
$\sin(b\,t)$	$\frac{b}{s^2+b^2}$

Formulae
$\mathcal{L}\left(e^{at}f(t)\right) = F(s-a)$
$\mathcal{L}\left(f'(t)\right) = sF(s) - f(0)$
$\mathcal{L}\left(u(t-a)f(t-a)\right) = e^{-as}F(s-a)$
$\mathcal{L}\left(tf(t)\right) = -F'(s)$